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Program for Solving Assignment Problems and Its Application in Lecturer Resources Allocation

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N Wattanasiripong[1] and N W Sangwaranatee[2,*]

[1]Applied Mathematics, Faculty of Science and Technology,
Valaya Alongkorn Rajabhat University, Pathumthani 13180, Thailand
[2]Informatics Mathematics Research Group, Faculty of Science and Technology,
Suan Sunandha Rajabhat University, Bangkok 10300, Thailand

[*]Corresponding author: nisakorn.su@ssru.ac.th

Abstract. This paper proposes program for solving assignment problems by Hungarian method. Assignment problem is one of the most famous problems in linear programming and in combinatorial optimization. It can be solved by using an efficient method which is called Hangarian method. In this problem, commonly, there are a number of agents and a number of tasks. Any agent can be assigned to perform any task. The objective of this problem is to decide the right agent to perform the right task and also the total cost of the assignment is minimized. The manual calculation is time-consuming and prone to mistake. Therefore, this program can help to find the optimal solution with less time. In this approach, computer programming is used as an implementation tool to get the accurate solutions. The program can also be applied with the lecturer allocation for each subject to minimize lecturer's preparation time and to match the lecturer per individual expertise.

Key words: Program Development, Hungarian Method, Linear Programming, Optimal Solution

1. Introduction

The problem on resource assignment or task assignment [1-4] for equipment, an individual employee, or a team to have proper responsibility is one of the critical decisions which should be made to maximize efficiency, to deliver the best result for organization including to minimize cost and expense, maximize productivity and sales. Therefore, the management should realize the problems and be thoughtful to gather information and solve them based on data and not being bias in decision making. Focusing on task assignment may create soft issue among the team, so the quantitative analysis technique can also help optimizing equipment location, plant layout and production plan.

The Hungarian method [5] is one of the widely used problem solving tools, Kuhn [6], Xian-Ying [7], that is used to allocate lecturers to minimize their class preparation time, and match lecturers' expertise at the same time. Kabiru et al. [8] used the Hungarian Method using LINGO program to optimize science lecturers in Nigeria. O. Solaja et al. [9] maximized teaching efficiency of university lecturers in Nigeria using Hungarian Method.

The research applied knowledge in mathematics and information technology to develop program to assign tasks using Hungarian Method that is effective and efficient. The in-house development program is expected to deliver the result – fit for purpose, convenient, and efficient - as same as a the licensed one to minimize class preparation time so that all the lecturers can spend more time on researches and projects.

2. Task Assignment Mathematical Model

Task assignment model is a type of linear programming models [10] which uses to allocate equipment, manpower and cost to improve production or services which lead to improve profitability, reduce cost, and/or maximize efficiency.

2.1 General characteristics of task assignment model

2.1.1 The number of resource and task must be the same, equals to n.

2.1.2 The pair of resource and task must be in a one-to-one relationship. Each resource is mutually exhausted, and all must be paired.

2.1.3 Fitness Function must be developed to seek goal with highest or lowest number such as cost, time, satisfaction, etc.

This is called the Balanced Assignment Problem which can be written in form of decision-making model. For example, the task assignment has a task number and a number of team member equal to "n", with the objective to minimize cost. The information and variables are as follow:

c_{ij} = cost of assign team member *i* on task *j*

x_{ij} = binary variable which either has a value of 0 or 1 which:

$x_{ij} = 1$ if team member *i* is assigned for task *j*, and

$x_{ij} = 0$ if team member *i* is not assigned for task *j*,

for $i = 1, 2, ..., n$ and $j = 1, 2, ..., n$

From the above example, the cost minimization equation can be written as follow:

To find the smallest number of $Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$ (1)

Within the condition of $\sum_{i=1}^{n} x_{ij} = 1$ for $j = 1, 2, ..., n$ (Each person performing a task) (2)

$\sum_{j=1}^{n} x_{ij} = 1$ for $i = 1, 2, ..., n$ (Each task performed by one person) (3)

$x_{ij} = 1$ or 0 for any *i* and *j* (4)

Where n is the number of person or task

The task assignment that is not balanced, the number of tasks and the number of person or equipment performing the task are different, requires a dummy task or person to equalize both numbers. The cost of each dummy has an infinite value (∞).

The calculation of decision variable x_{ij} can be done by using simplex method that produce an integer result or by applying a model used to solve transportation problem. However, the application of both methods is not widely used due to their complexity. Then, the Hungarian mathematicians named, D. Konig, then Kuhn [5-6], invented Hungarian Method to manage task assignment on the following theory.

Theorem 1: The decision variable x_{ij} remains constant if all constants in c_{ij} changes equally in the matrix or expense table. theorem 1 defines the task assignment problem referring to the equation (1)-(4);

If the set of binary variables $\{x_{ij}\}$ is the set that provides the lowest $z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$, the set of binary variables $\{x_{ij}\}$ will also provide the lowest $z' = \sum_{i=1}^{n}\sum_{j=1}^{n} c'_{ij}x_{ij}$ when $c'_{ij} = u_i + v_j - c_{ij}$ with constant $u_i, v_j, i = 1,2,...,n$ and $j = 1,2,...,n$.

Thus,
$$\begin{aligned} z' &= \sum_{i=1}^{n}\sum_{j=1}^{n}\left(u_i + v_j - c_{ij}\right)x_{ij} \\ &= -\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{i=1}^{n} u_i \sum_{j=1}^{n} x_{ij} + \sum_{j=1}^{n} v_i \sum_{i=1}^{n} x_{ij} \\ &= \sum_{i=1}^{n} u_i + \sum_{j=1}^{n} v_i - z \end{aligned}$$

Since the terms $\sum_{i=1}^{n} u_i$ and $\sum_{j=1}^{n} v_i$ are constant and independent of all x_{ij}, minimizing z' will also minimize z regardless of $\sum_{i=1}^{n} u_i$ and $\sum_{j=1}^{n} v_i$ Therefore, the set of binary variables $\{x_{ij}\}$, or task assignment to minimize z is the same as to minimize z'.

Theorem 2 defines the task assignment (1)-(4), if all constants $c_{ij} \geq 0$ will have $x_{ij} \geq 0$ where ($i = 1,2,...,n$ and $j = 1,2,...,n$), will have the following objective function $z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} = 0$. Therefore, these x_{ij} is the optimal solution.

In the theorem 2, the set of binary variables $\{x_{ij}\}$ is the optimal solution. Therefore, the solution under the Hungarian Method is produced by adding or subtracting constant in each row and column until c_{ij} equals to 0. The Hungarian Method has the following steps.

2.2 The Step of Applying Hungarian Method to Standard Task Assignment (Minimize)

1. Find the smallest number of c_{ij} (can be negative) in each row to deduct all numbers in the same row. Then, find the smallest number of c_{ij} (can be negative) in each column to deduct all numbers in the same column. The outcome is the Reduced Matrix.
2. From the Reduced Matrix, draw the shortest vertical and horizontal lines that cross all 0 at least once. If the number of crossing equals to the number of row or column, then it gives the optimal result and ends the process. If the number of crossing is smaller, go to step 3.
3. Find more cells with zero number by selecting the number in the matrix that does not cross the line. Subtract other cells that do not cross the line with this value, and add this value back to all cells that have the two lines crossed. This step is to have all cells in matrix do not have negative value. Then, go back to step 2.

<u>Remark</u>: If the objective function is to find the maximum value, this method can be applied by multiplying -1 to call c_{ij} in the matrix. Then, follow the same steps to find the minimum value.

3. Working process of task assignment program

The task assignment program coding was developed by Java Language. The home page has 4 parts;

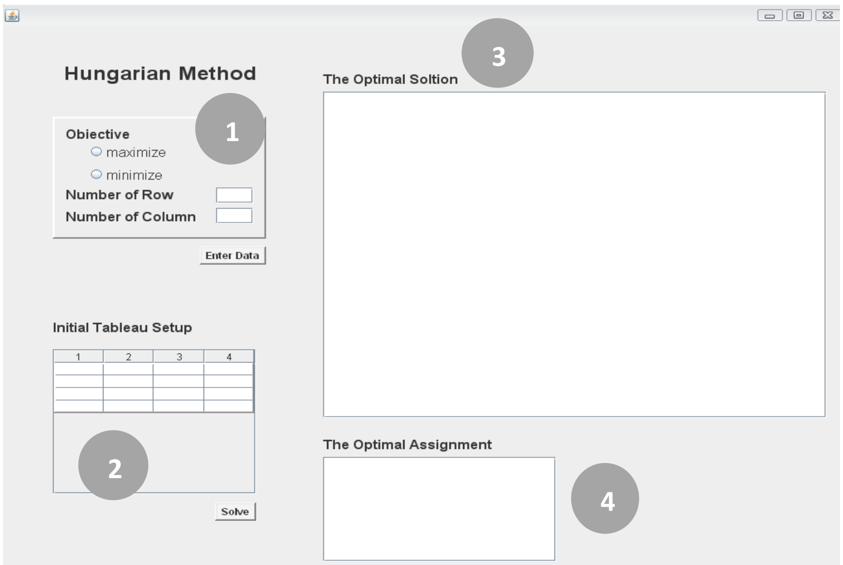

Figure 1. Home page to represent task assignment program.

1. The first part is the objective function with number of row and columns as inputs. It can be selected either to calculate the minimum or maximum value. The number of row and column define number of manpower and tasks.
2. The second part is the table showing the size of the calculation
3. The third part shows the steps of detailed calculations
4. The fourth part is the output of the calculation showing the optimal assignment

The program can solve for the minimum and maximum value without matrix size restriction. The numbers of team member and task can be different. It can also add dummy task or person into the calculation and send error message if input is incorrect. It can also handle the input with negative number.

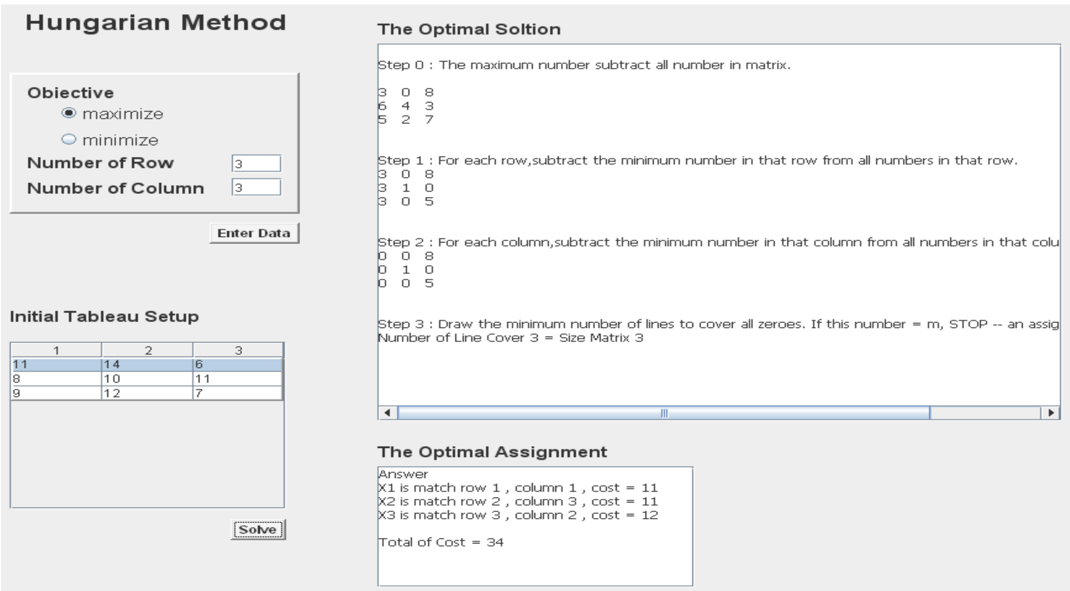

Figure 2. the example of task assignment finding maximum objective value

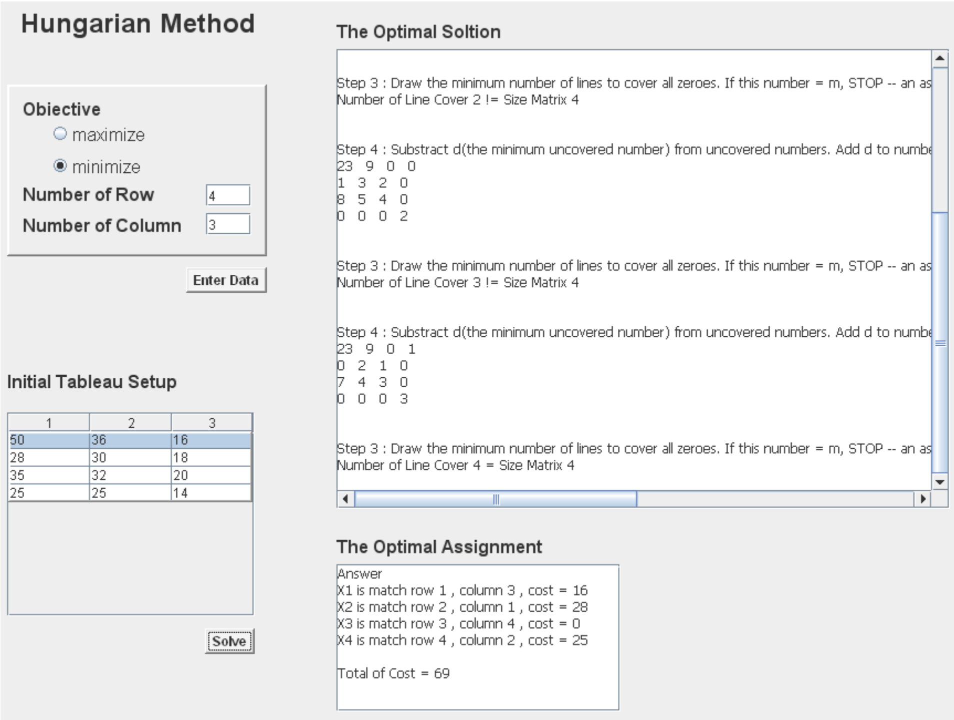

Figure 3. the example of task assignment with different number of task and person

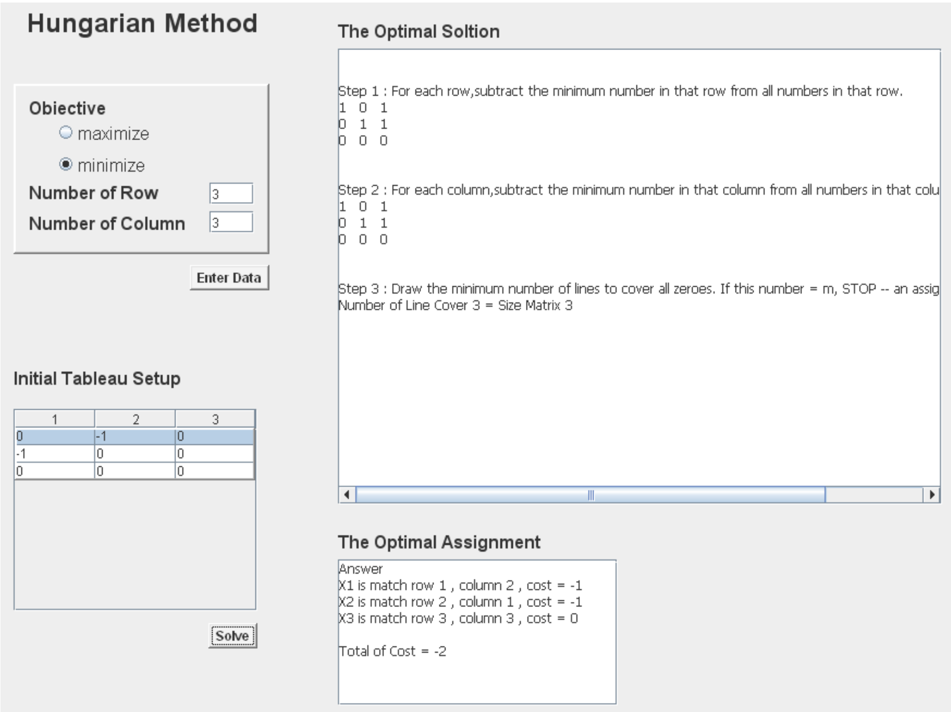

Figure 4. the example of task assignment with negative-value input

4. Case Analysis: Task assignments to minimize class preparation time for each lecturer

There are 4 mathematics lecturers who teach in general education program in one of the universities in Thailand. Each lecturer can cover 4 different subjects. With different individual experiences, each lecturer has different class-preparation duration as shown in Table 1. To allow enough time for lecturer to do other researches, each of them can give lecture only one subject. The question is how to assign each lecturer to minimize overall class preparation duration.

Table 1. Assignment model table formulation

Subject / Lecturer	**1 (days)**	**2 (days)**	**3 (days)**	**4 (days)**
A	15	18	18	16
B	14	19	13	17
C	11	16	13	14
D	12	16	14	15

Create linear equation variable as follow:

Given that x_{ij} = Match lecturer i with subject j

i = 1, 2, 3, or 4 representing lecturer A, B, C, or D respectively

j = 1, 2, 3, or 4 representing subject 1, 2, 3, or 4 respectively

Minimize the preparation time for lessons

$$Z = 15x_{11} + 18x_{12} + 18x_{13} + 16x_{14} + 14x_{21} + 19x_{22} + 13x_{23} + 17x_{24} + 11x_{31} + 16x_{32} + 13x_{33} + 14x_{34} + 12x_{41} + 16x_{42} + 14x_{43} + 15x_{44}$$

Subject to

(Each lecturer for one subject) (Each subject for one lecturer)

$$x_{11} + x_{12} + x_{13} + x_{14} = 1 \qquad x_{11} + x_{21} + x_{31} + x_{41} = 1$$

$$x_{21} + x_{22} + x_{23} + x_{24} = 1 \qquad x_{12} + x_{22} + x_{32} + x_{42} = 1$$

$$x_{31} + x_{32} + x_{33} + x_{34} = 1 \qquad x_{13} + x_{23} + x_{33} + x_{43} = 1$$

$$x_{41} + x_{42} + x_{43} + x_{44} = 1 \qquad x_{14} + x_{24} + x_{34} + x_{44} = 1$$

$$x_{ij} = 0, 1 \quad i = 1, 2, 3, 4\ ,\ j = 1, 2, 3, 4$$

This scenario has the objective to find the minimum value of the preparation time for lessons for 4 lecturers with 4 subjects. Insert inputs in the table into the task assignment program and choose minimize objective because the desired result is the lecturer assignment for each subject will produce the least amount of preparation time. There are 4 lecturers (row) and 4 subjects (columns). Click solve button then the program will calculate the optimal solution by using Hungarian Method.

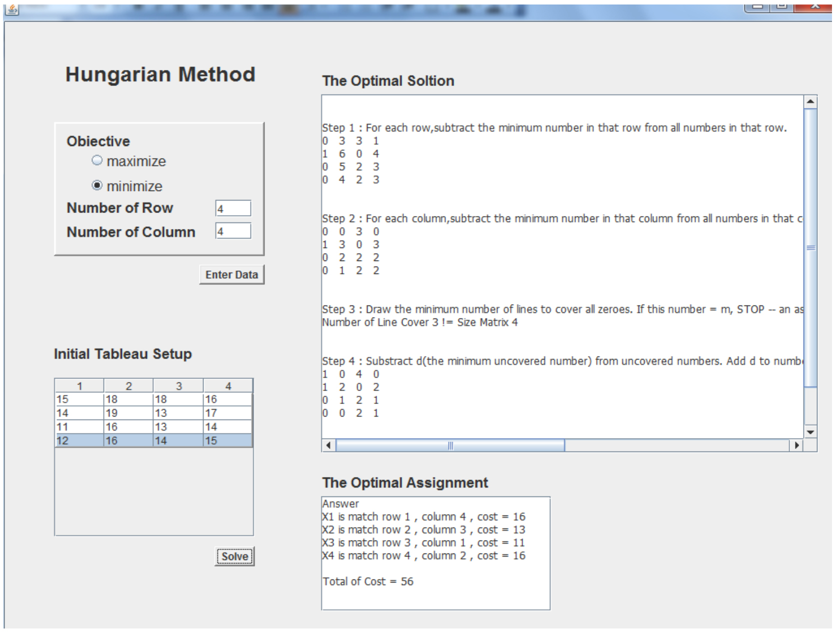

Figure 5. shows the case analysis result of lecturer assignment

Table 2. Optimum allocation table

Subjects	Lecturer	Days for preparing
1	C	11
2	D	16
3	B	13
4	A	16
Total for preparing		56

Table 2 above shows the result of courses allocation obtained with the help of Hungarian method. The results suggest that the department should allocate Subject 1 to a lecturer C, Subject 2 to a lecturer D, Subject 3 to a lecturer B and Subject 1 to a lecturer A. That they will use hours to prepare 11, 16. 13 and 16, respectively.

5. Conclusion

The research applies knowledge in Mathematics and Information Technology to develop a task assignment program by Hungarian Method in Java language. The program demonstrates steps of Hungarian Method and efficiently provides result. It can be used in real situation to allocate mathematics lecturers to other subjects with minimum class preparation time, so that each lecturer can spend their time on other useful researches for the university.

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