# Tourist tram routing problem in Lamphun City, Thailand 

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#### Abstract

This research aims to study the tourist tram routing problem in Lamphun City, Thailand. Considered an application of the Travelling Salesman Problem (TSP), the distance data along all possible routes were gathered from Google Maps and Google Earth, including additional information provided by the city. First, the shortest distance between each pair of the tourist spots is determined. Then, the linear programming (LP) model for TSP is employed so as to find the best tourist tram route traversing through all planned tourist spots with minimum total distance. The solution obtained from this model is then compared with those from the nearest neighbor and the savings algorithms for TSP. The comparison shows that the optimal tram routes from the LP model and the savings algorithm are, though different, tied at the minimum total distance of 9,770 meters, reduced by $9.36 \%$ from the


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current 10,779-meter route. This optimal route is also better than the of 10,050-meter route obtained from the nearest neighbor algorithm.

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## 1. Introduction

Recently, Lamphun, a northern city of Thailand, was chosen by Tourism Authority of Thailand (TAT) to be a must-not-miss city for tourists, Thai and foreigners alike. It connects tourists to Lampang, Chiang Mai and Chiang Rai. Lamphun is well-known for its simplicity and tranquility with beautiful arts, cultures, and traditions. They can easily be perceived through every tourist attraction such as temples, ancient Chiapas, ancient buildings, and heritage sites. The embedded 1,340-year-old civilization of the ancient Lanna Kingdom helps bring visitors to Lamphun continuously. Tourism development strategies have also been launched for the city. One such example is that the tour buses have been transferred from the Lamphun Tourist and Sports Office to the Lamphun Municipality.

Another example, being also the focus of this research, is the tourist tram service that has been provided throughout 11 major tourist spots in Lamphun. They will later be stated and worked for. This tram service is claimed to be "the tour of Lamphun", the city of Boon Luang (translated to "royal merit") of the ancient Lanna. The tour takes approximately 2-3 hours. The tram operates for only 2 rounds a day, at 9.30 and 13.30.

The current tourist tram routing arrangement of Lamphun City only concerns taking tourists to every attraction. No algorithms or models are used in assisting the route planning. Based on the characteristics of this tram routing problem, getting started from the Hiripunchai Temple, going through 11 tourist spots, and returning to the Hiripunchai Temple, this problem is, in nature, the Travelling Salesman Problem (TSP) without an objective.

Therefore, in this research, the TSP model is incorporated with an objective of finding the best tram route having minimum total travel distance, as part of the green vehicle routing problem [1]. The model is subsequently be solved by Solver in Excel and the solution is then compared with those from other approaches, namely, the savings algorithm and the nearest neighbor algorithm. For clarification, because the tram's travel
time depends on some uncontrollable factors such as the tram speed and the route's traffic, the travel time are simply disregarded.

The rest of this article is organized as follows. A review on theories and literature is presented next. The procedure steps used in this research are explained in Section 3, followed by the results and discussion in Section 4. Finally, a conclusion of this work is drawn in Section 5.

## 2. Theory and Literature Review

To find the optimal tourist tram route, TSP, the nearest neighbor algorithm, and the savings algorithm are adapted and compared. Theoretical background and relevant literature are reviewed here. Starting with TSP as follows.

TSP can be formulated as a linear programming or LP model [14] and solved by Microsoft Excel Solver accordingly. A set of notations used is defined here in the context of this tourist tram problem. It is then followed by the LP model for TSP, as follows.

## Parameters:

$V$ : set of all tourist spots
$i, j$ : tourist spot $i$ or $j ; i, j=0,1,2,3, \ldots, k-1$
$c_{i j}$ : distance between tourist spot $i$ and $j$
$k$ : number of tourist spots
$S$ : set of all possible routes

## Decision-variables:

$x_{i j}=1$ if spot $j$ is visited immediately after $i$
$x_{i j}=0$ otherwise

## Objective function

Minimize $\quad \sum_{i \neq j} c_{i j} x_{i j}$
Subject to

$$
\begin{equation*}
\sum_{j=1}^{k} x_{i j}=1 ; i=0,1,2,3, \ldots, k-1 \tag{2}
\end{equation*}
$$

$$
\begin{gather*}
\sum_{i=1}^{k} x_{i j}=1 ; j=0,1,2,3, \ldots, k-1  \tag{3}\\
\sum_{i, j \in S}^{k} x_{i j} \leq|S|-1 \quad S \subset V, 2 \leq|S| \leq k-2 \tag{4}
\end{gather*}
$$

Equation (1): The objective function aims to minimize the total traveling distance of the tram route

Equation (2): After spot $i$, only one next spot is visited.
Equation (3): Before spot $j$, only one previous spot is visited.
Equation (4): This is to prevent a sub-tour.

Besides the LP model for TSP, this tourist tram routing problem can be solved by the nearest neighbor algorithm according to the following steps.

1. Start at the initial tourist spot $i$, in this case, the Hiripunchai Temple and mark as "visited".
2. Select the next "unvisited" tourist spot $j$ having the shortest distance from the previously visited spot $i$.
3. Mark spot $j$ as "visited" and reset spot $i=j$.
4. Go back to step 2 until all the tourist spots are marked as "visited"

The route can be obtained by connecting all the visited tourist spots sequentially, and then connecting the last spot back to the starting spot. Sum all the distances in the route to obtain the total distance.

The last algorithm that we use to search for the best tourist tram route is the savings algorithm [5]. It is one of the most common heuristics used for small-and-medium-scaled problems. The steps of the algorithm are as follows.

1. Select any tourist spot as the center and number it as 1 .
2. Compute the savings cost $s_{i j}=c_{1 i}+c_{1 j}-c_{i j}$ for $i, j=2,3, \ldots, n$.
3. Order the savings cost in descending order.
4. Looking down from the top of the savings list, form larger sub-tours by connecting proper tourist spots $i$ and $j$. Repeat until the route is fully formed.

Previous studies on TSP include the following. [6] finds the shortest route for the water meter officials of the Prachin Buri Provincial Waterworks Authority to travel by car to measure the water meters in the area of Prachin Buri province. The resulting new route yield a lower total travel distance than the current route. Similarly, [7] adapts TSP for planning optimal travel routes in a pearl milk tea business. The TSP-based program [8] developed for solving this case study shows that their overall transportation costs can be reduced by $18.15 \%$.

As for the savings algorithms, it is used in [9] to determine the optimal number of shipping trucks in retail businesses. Their case study of Top Supermarket shows that at the optimal number of trucks of 59, it can save the transportation costs by $4.22 \%$.

In another application, [10] tries to relocate the flood victims out of the disastrous areas by bus. The savings algorithm produces the shortest bus route of 909.39 km .

Also, [11] compares the TSP model with the savings algorithm on planning the route for delivering drinking water from the plant to the customers according to the partitioned service areas. The results show that the optimal route obtained from the TSP model [12-13] is $4.16 \%$ shorter than that from the savings algorithms.

## 3. Research Procedure

In this section, the research steps to carry out experiments for finding the optimal tram route in Lamphun City are as follows.

Step 1 (Data Collection): Collect the data regarding all the tourist spots along the route. Google Maps and surveying the real locations are the data collection methods. As a result, 11 tourist spots together with 26 intersections are identified. Figure 1 shows the current Lamphun City tram route visualized on Google Maps.

As the input data for this research, the 11 tourist spots are labeled and numbered as Nodes $1,2, \ldots, 11$ according to the following list:

1. Phra That Hariphunchai Temple
2. Lamphun Community Museum
3. Khum Chao Yod-Ruen Community Center
4. Phra-Nang Cham Dhevi
5. Cham Dhevi temple
6. Maha Wan Temple
7. Phakhong Ruesi temple
8. Pa Yang Luong Temple
9. Goochang Gooma Historical Site
10. Phra Yuen Temple
11. Phrakaew Temple

Also, the 26 intersections are labels as Nodes A to Z. Figure 2 shows the collected direct distances between all the nodes.


Figure 1
The current Lamphun City tram route on Google Maps.


Figure 2
The shortest distances between the tourist spots and intersections

Step 2 (Data Preparation): To prepare the input data for TSP, the shortest distance between each pair of the tourist spots is calculated by formulating as the shortest path problem with the direct distances from Figure 2 as the input data.

Step 3 (LP Formulation for TSP): Formulate this tram routing problem as an LP model for TSP using the shortest distances between tourist spots from step 2 and then solve the model by Excel Solver.

Step 4 (Applying the Algorithms): First, apply the nearest neighbor algorithm to this tram routing problem. Then, apply the savings algorithm as well.

Step 5 (Result Comparison): Compare the current tram route with the routes obtained from the LP model, the nearest neighbor algorithm, and the savings algorithm.

## 4. Results and Discussion

Results from each research procedure step are presented and discussed in this section.

Step 1 (Data Collection): To be able to compare the results of all algorithms in this research, the current tourist tram route having no assistance from any methods is first identified as follows: $1-\mathrm{A}-\mathrm{B}-\mathrm{C}-\mathrm{D}-2-\mathrm{E}-3-\mathrm{H}-$ $\mathrm{I}-4-6-5-6-\mathrm{L}-7-\mathrm{Q}-\mathrm{P}-8-\mathrm{R}-9-\mathrm{S}-\mathrm{U}-10-\mathrm{U}-\mathrm{T}-11-\mathrm{O}-\mathrm{A}-1$

This current tram route has a total distance of 10,779 meters. Also, when the intersections are removed from the list, the sequence of the tourist spots in the current route is simply $1,2,3,4,6,5,6,7,8,9,10,11,1$. This route is visualized in Figure 3.

Step 2 (Data Preparation): The shortest distance between every pair of tourist spots is identified by solving each associated shortest path problem. Such a shortest path problem is finding a shortest path from tourist spot 1 to 11. It can be formulated as a Linear Programming model as follows.

Let $x_{i j}$ be the direct path between $i$ and $j$
where $i, j=1,2,3,4,5,6,7,8,9,10,11, A, B, C, D, E, F, G, H, I, J, K, L, M, N$, $O, P, Q, R, S, T, V, W, X, Y, Z$
$x_{i j}=1$ if this $i$ - $j$ line is chosen in the path
$x_{i j}=0$ if this $i-j$ line is not chosen in the path


Figure 3
The current route of the Lamphun City Tram
Then, he objective of this problem is minimize the total distance of the path from Node 1 to Node 11 by deciding whether $x_{i j}$ should be included in the path.

In other word, the objective function can be expressed as
minimize $210 x_{1 A}+168 x_{1 W}+210 x_{A 1}+180 x_{A B}+245 x_{A W}+280 x_{A O}+350 x_{A N}$
$+480 x_{A M}+180 x_{B A}+100 x_{B C}+100 x_{C B}+50 x_{C D}+65 x_{C E}+50 x_{D C}+59 x_{D F}$
$+105 x_{D 2}+59 x_{F D}+280 x_{F W}+85 x_{F G}+160 x_{F J}+65 x_{E C}+65 x_{E 2}+100 x_{E 3}+65 x_{2 E}$
$+105 x_{2 D}+40 x_{2 G}+85 x_{G F}+40 x_{G 2}+190 x_{G I}+160 x_{J F}+300 x_{J K}+207 x_{J V}+100 x_{3 E}$
$+35 x_{3 H}+35 x_{H 3}+50 x_{H I}+50 x_{I H}+190 x_{I G}+450 x_{I 4}+140 x_{I 6}+800 x_{65}+140 x_{6 I}$
$+745 x_{6 L}+800 x_{56}+450 x_{4 I}+1080 x_{46}+350 x_{4 V}+350 x_{V 4}+207 x_{V J}+300 x_{K J}$
$+180 x_{K W}+480 x_{K X}+245 x_{W A}+280 x_{W F}+168 x_{W 1}+180 x_{W K}+350 x_{N A}+477 x_{N P}$
$+78 x_{N O}+25 x_{M L}+745 x_{L 6}+200 x_{L N}+25 x_{L M}+185 x_{L 7}+185 x_{7 L}+400 x_{7 Q}$
$+100 x_{8 P}+60 x_{8 R}+400 x_{Q 7}+280 x_{Q P}+280 x_{P Q}+477 x_{P N}+100 x_{P 8}+60 x_{R 8}$
$+340 x_{R Z}+555 x_{R S}+1090 x_{R 9}+340 x_{Z R}+390 x_{Z 9}+1090 x_{9 R}+390 x_{9 Z}+600 x_{9 S}$
$+600 x_{S 9}+555 x_{S R}+800 x_{S O}+840 x_{S U}+800 x_{O S}+280 x_{O A}+223 x_{O Y}+635 x_{O, 11}$
$+223 x_{Y O}+685 x_{Y T}+325 x_{Y, 11}+325 x_{11, Y}+635 x_{11, O}+447 x_{11, T}+480 x_{X K}$
$+520 x_{X, 11}+1050 x_{X U}+685 x_{T Y}+447 x_{T, 11}+302 x_{T U}+840 x_{U S}+302 x_{U T}$
$+1050 x_{u x}+290 x_{u, 10}+290 x_{10, u}$
subject to the following balance constraints:

$$
\begin{gathered}
\sum_{j=1}^{11} x_{1 j}-\sum_{j=1}^{11} x_{j 1}=1 \\
\sum_{j=1}^{11} x_{i j}-\sum_{j=1}^{11} x_{j i}=1 \text { for } i=2,3, \ldots, 10 \\
\sum_{j=1}^{11} x_{j, 11}-\sum_{j=1}^{11} x_{11, j}=1
\end{gathered}
$$

and $x_{i j}=0$ or 1

When this model is entered to Excel to be solved by Solver, the shortest distance between every pair of the tram tourist spots is then obtained and reported here in Table 1.

Table 1
The shortest distance between every pair of the tourist spots in Lamphun City

| $i, j$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | 573 | 655 | 1,165 | 1,663 | 863 | 900 | 1,137 | 1,890 | 1,990 | 1,038 |
| 2 | 573 |  | 165 | 680 | 1,170 | 370 | 1,100 | 1,337 | 2,090 | 2,190 | 1,238 |
| 3 | 655 | 165 |  | 535 | 1,025 | 225 | 1,135 | 1,372 | 2,125 | 2,225 | 1,273 |
| 4 | 1,165 | 680 | 535 |  | 1,390 | 590 | 1,520 | 1,907 | 2,660 | 2,677 | 1,808 |
| 5 | 1,663 | 1,170 | 1,025 | 1,390 |  | 800 | 1,730 | 2,322 | 3,112 | 3,250 | 2,298 |
| 6 | 863 | 370 | 225 | 590 | 800 |  | 930 | 1,522 | 2,312 | 2,450 | 1,498 |
| 7 | 945 | 1,145 | 1,155 | 1,520 | 1,730 | 930 |  | 780 | 1,570 | 1,963 | 1,011 |
| 8 | 1,137 | 1,337 | 1,372 | 1,907 | 2,397 | 1,597 | 780 |  | 790 | 1,745 | 1,203 |
| 9 | 1,890 | 2,090 | 2,125 | 2,660 | 3,150 | 2,350 | 1,570 | 790 |  | 1,730 | 1,948 |
| 10 | 1,990 | 2,190 | 2,225 | 2,677 | 3,250 | 2,450 | 2,470 | 1,745 | 1,730 |  | 1,039 |
| 11 | 1,038 | 1,238 | 1,273 | 1,808 | 2,298 | 1,498 | 1,518 | 1,755 | 1,948 | 1,039 |  |

These distances in Table 1 will then be used as initial data for the TSP model and algorithms in the next steps.

Step 3 (LP Formulation for TSP): Since this tram routing problem can be viewed as TSP, the LP model can thus be formulated using the shortest distances between tourist spots from step 2 so to minimize the total traveling distance along the route as follows.

$$
\begin{aligned}
& \text { Minimize } 573 x_{1,2}+655 x_{1,3}+1165 x_{1,4}+1663 x_{1,5}+863 x_{1,6}+900 x_{1,7} \\
& +1137 x_{1,8}+1890 x_{1,9}+1990 x_{1,10}+1038 x_{1,11}+573 x_{2,1}+165 x_{2,3}+680 x_{2,4} \\
& +1170 x_{2,5}+370 x_{2,6}+1100 x_{2,7}+1337 x_{2,8}+2090 x_{2,9}+2190 x_{2,10}+1238 x_{2,11} \\
& +655 x_{3,1}+165 x_{3,2}+535 x_{3,4}+1025 x_{3,5}+225 x_{3,6}+1135 x_{3,7}+1372 x_{3,8} \\
& +2125 x_{3,9}+2225 x_{3,10}+1273 x_{3,11}+1165 x_{4,1}+680 x_{4,2}+535 x_{4,3}+1390 x_{4,5} \\
& +590 x_{4,6}+1520 x_{4,7}+1907 x_{4,8}+2660 x_{4,9}+2677 x_{4,10}+1808 x_{4,11}+1663 x_{5,1} \\
& +1170 x_{5,2}+1025 x_{5,3}+1390 x_{5,4}+800 x_{5,6}+1730 x_{5,7}+2322 x_{5,8}+3112 x_{5,9} \\
& +3250 x_{5,10}+2298 x_{5,11}+863 x_{6,1}+370 x_{6,2}+225 x_{6,3}+590 x_{6,4}+800 x_{6,5} \\
& +930 x_{6,7}+1522 x_{6,8}+2312 x_{6,9}+2450 x_{6,10}+1498 x_{6,11}+945 x_{7,1}+1145 x_{7,2} \\
& +1155 x_{7,3}+1520 x_{7,4}+1730 x_{7,5}+930 x_{7,6}+780 x_{7,8}+1570 x_{7,9}+1963 x_{7,10} \\
& +1011 x_{7,11}+1137 x_{8,1}+1337 x_{8,2}+1372 x_{8,3}+1907 x_{8,4}+2397 x_{8,5}+1597 x_{8,6} \\
& +780 x_{8,7}+790 x_{8,9}+1745 x_{8,10}+1203 x_{8,11}+1890 x_{9,1}+2090 x_{9,2}+2125 x_{9,3} \\
& +2660 x_{9,4}+3150 x_{9,5}+2350 x_{9,6}+1570 x_{9,7}+790 x_{9,8}+1730 x_{9,10}+1948 x_{9,11} \\
& +1990 x_{10,1}+2190 x_{10,2}+2225 x_{10,3}+2677 x_{10,4}+3250 x_{10,5}+2450 x_{10,6} \\
& +2470 x_{10,7}+1745 x_{10,8}+1730 x_{10,9}+1039 x_{10,11}+1038 x_{11,1}+1238 x_{11,2} \\
& +1273 x_{11,3}+1808 x_{11,4}+2298 x_{11,5}+1498 x_{11,6}+1518 x_{11,7}+1755 x_{11,8} \\
& +1948 x_{11,9}+1039 x_{11,10}
\end{aligned}
$$

subject to the following constraints:
Constraint Group 1: After spot $i$, only one next spot is visited.
Node $1 \quad x_{1,2}+x_{1,3}+x_{1,4}+x_{1,5}+x_{1,6}+x_{1,7}+x_{1,8}+x_{1,9}+x_{1,10}+x_{1,11}=1$
Node $2 x_{2,1}+x_{2,3}+x_{2,4}+x_{2,5}+x_{2,6}+x_{2,7}+x_{2,8}+x_{2,9}+x_{2,10}+x_{2,11}=1$
Node $3 x_{3,1}+x_{3,2}+x_{3,4}+x_{3,5}+x_{3,6}+x_{3,7}+x_{3,8}+x_{3,9}+x_{3,10}+x_{3,11}=1$
Node $4 \quad x_{4,1}+x_{4,2}+x_{4,3}+x_{4,5}+x_{4,6}+x_{4,7}+x_{4,8}+x_{4,9}+x_{4,10}+x_{4,11}=1$
Node $5 x_{5,1}+x_{5,2}+x_{5,3}+x_{5,4}+x_{5,6}+x_{5,7}+x_{5,8}+x_{5,9}+x_{5,10}+x_{5,11}=1$
Node $6 x_{6,1}+x_{6,2}+x_{6,3}+x_{6,4}+x_{6,5}+x_{6,7}+x_{6,8}+x_{6,9}+x_{6,10}+x_{6,11}=1$
Node $7 \quad x_{7,1}+x_{7,2}+x_{7,3}+x_{7,4}+x_{7,5}+x_{7,6}+x_{7,8}+x_{7,9}+x_{7,10}+x_{7,11}=1$
Node $8 \quad x_{8,1}+x_{8,2}+x_{8,3}+x_{8,4}+x_{8,5}+x_{8,6}+x_{8,7}+x_{8,9}+x_{8,10}+x_{8,11}=1$
Node $9 \quad x_{9,1}+x_{9,2}+x_{9,3}+x_{9,4}+x_{9,5}+x_{9,6}+x_{9,7}+x_{9,8}+x_{9,10}+x_{9,11}=1$

Node $10 x_{10,1}+x_{10,2}+x_{10,3}+x_{10,4}+x_{10,5}+x_{10,6}+x_{10,7}+x_{10,8}+x_{10,9}+x_{10,11}=1$
Node $11 x_{11,1}+x_{11,2}+x_{11,3}+x_{11,4}+x_{11,5}+x_{11,6}+x_{11,7}+x_{11,8}+x_{11,9}+x_{11,10}=1$

Constraint Group 2: Before spot $j$, only one previous spot is visited.
Node $1 \quad x_{2,1}+x_{3,1}+x_{4,1}+x_{5,1}+x_{6,1}+x_{7,1}+x_{8,1}+x_{9,1}+x_{10,1}+x_{11,1}=1$
Node $2 x_{1,2}+x_{3,2}+x_{4,2}+x_{5,2}+x_{6,2}+x_{7,2}+x_{8,2}+x_{9,2}+x_{10,2}+x_{11,2}=1$
Node $3 x_{1,3}+x_{2,3}+x_{4,3}+x_{5,3}+x_{6,3}+x_{7,3}+x_{8,3}+x_{9,3}+x_{10,3}+x_{11,3}=1$
Node $4 \quad x_{1,4}+x_{2,4}+x_{3,4}+x_{5,4}+x_{6,4}+x_{7,4}+x_{8,4}+x_{9,4}+x_{10,4}+x_{11,4}=1$
Node $5 \quad x_{6,5}=1$
Node $6 x_{1,6}+x_{2,6}+x_{3,6}+x_{4,6}+x_{5,6}+x_{7,6}+x_{8,6}+x_{9,6}+x_{10,6}+x_{11,6}=1$
Node $7 \quad x_{1,7}+x_{2,7}+x_{3,7}+x_{4,7}+x_{5,7}+x_{6,7}+x_{8,7}+x_{9,7}+x_{7,10}+x_{7,11}=1$
Node $8 \quad x_{1,8}+x_{2,8}+x_{3,8}+x_{4,8}+x_{5,8}+x_{6,8}+x_{7,8}+x_{9,8}+x_{10,8}+x_{11,8}=1$
Node $9 \quad x_{1,9}+x_{2,9}+x_{3,9}+x_{4,9}+x_{5,9}+x_{6,9}+x_{7,9}+x_{8,9}+x_{10,9}+x_{11,9}=1$
Node $10 x_{1,10}+x_{2,10}+x_{3,10}+x_{4,10}+x_{5,10}+x_{6,10}+x_{7,10}+x_{8,10}+x_{9,10}+x_{11,10}=1$
Node $11 x_{1,11}+x_{2,11}+x_{3,11}+x_{4,11}+x_{5,11}+x_{6,11}+x_{7,11}+x_{8,11}+x_{9,11}+x_{10,11}=1$

## Constraint Group 3: Subtour Prevention.

$$
\begin{array}{lllll}
x_{1,2}+x_{2,1} \leq 1 & x_{1,3}+x_{3,1} \leq 1 & x_{1,4}+x_{4,1} \leq 1 & x_{1,5}+x_{5,1} \leq 1 & x_{1,6}+x_{6,1} \leq 1 \\
x_{1,7}+x_{7,1} \leq 1 & x_{1,8}+x_{8,1} \leq 1 & x_{1,9}+x_{9,1} \leq 1 & x_{1,10}+x_{10,1} \leq 1 & x_{1,11}+x_{11,1} \leq 1 \\
x_{2,3}+x_{3,2} \leq 1 & x_{2,4}+x_{4,2} \leq 1 & x_{2,5}+x_{5,2} \leq 1 & x_{2,6}+x_{6,2} \leq 1 & x_{2,7}+x_{7,2} \leq 1 \\
x_{2,8}+x_{8,2} \leq 1 & x_{2,9}+x_{9,2} \leq 1 & x_{2,10}+x_{10,2} \leq 1 & x_{2,11}+x_{11,2} \leq 1 & x_{3,4}+x_{4,3} \leq 1 \\
x_{3,5}+x_{5,3} \leq 1 & x_{3,6}+x_{6,3} \leq 1 & x_{3,7}+x_{7,3} \leq 1 & x_{3,8}+x_{8,3} \leq 1 & x_{3,9}+x_{9,3} \leq 1 \\
x_{3,10}+x_{10,3} \leq 1 & x_{3,11}+x_{11,3} \leq 1 & x_{4,5}+x_{5,4} \leq 1 & x_{4,6}+x_{6,4} \leq 1 & x_{4,7}+x_{7,4} \leq 1 \\
x_{4,8}+x_{8,4} \leq 1 & x_{4,9}+x_{9,4} \leq 1 & x_{4,10}+x_{10,4} \leq 1 & x_{4,11}+x_{11,4} \leq 1 & x_{5,6}+x_{6,5} \leq 1 \\
x_{5,7}+x_{7,5} \leq 1 & x_{5,8}+x_{8,5} \leq 1 & x_{5,9}+x_{9,5} \leq 1 & x_{5,10}+x_{10,5} \leq 1 & x_{5,11}+x_{11,5} \leq 1 \\
x_{6,7}+x_{7,6} \leq 1 & x_{6,8}+x_{8,6} \leq 1 & x_{6,9}+x_{9,6} \leq 1 & x_{6,10}+x_{10,6} \leq 1 & x_{6,11}+x_{11,6} \leq 1 \\
x_{7,8}+x_{8,7} \leq 1 & x_{7,9}+x_{9,7} \leq 1 & x_{7,10}+x_{10,7} \leq 1 & x_{7,11}+x_{11,7} \leq 1 & x_{8,9}+x_{9,8} \leq 1 \\
x_{8,10}+x_{10,8} \leq 1 & x_{8,11}+x_{11,8} \leq 1 & x_{9,10}+x_{10,9} \leq 1 & x_{9,11}+x_{11,9} \leq 1 & x_{10,11}+x_{11,10} \leq 1
\end{array}
$$

This LP model for TSP is then entered into Excel and solved by Solver. However, the solution obtained split the tram route into three parts, that is, $1-2-3-1,5-4-6-5$ and $7-11-10-9-8-7$.

Thus, it is necessary to add more constraints to eliminate these subtour as follows.

$$
\begin{gathered}
x_{1,2}+x_{2,3}+x_{3,1} \leq 2 \\
x_{1,3}+x_{3,2}+x_{2,1} \leq 2 \\
x_{5,4}+x_{4,6}+x_{6,5} \leq 2 \\
x_{5,6}+x_{6,4}+x_{4,5} \leq 2 \\
x_{7,11}+x_{11,10}+x_{10,9}+x_{9,8}+x_{8,7} \leq 4 \\
x_{7,8}+x_{8,9}+x_{9,10}+x_{10,11}+x_{11,7} \leq 4
\end{gathered}
$$

Then, this revised LP model is again solved by Solver. The optimal tram route is now in one piece, that is, $1-2-3-4-6-5-7-8-9-10-11-1$ and a total distance of the route is 9,770 meters.

Step 4 (Applying the Algorithms): In this step, both the nearest neighbor and the savings algorithms will be applied to this tourist tram routing problem.

By the nearest neighbor algorithm, starting with tourist spot 1, from Row 1 in Table 2, obviously, the closest one is spot 2 with a distance of 573 meters. Then, look at Row 2, the smallest number, 573 excluded, is 165

Table 2
The Nearest Neighbor Algorithm Process.

| $i, j$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | 573 | 655 | 1165 | 1683 | 863 | 900 | 1137 | 1890 | 1990 | 1038 |
| 2 | 573 |  | 165 | 680 | 1170 | 370 | 1100 | 1337 | 2090 | 2190 | 1238 |
| 3 | 655 | 165 |  | 535 | 1025 | 225 | 1135 | 1372 | 2125 | 2225 | 1273 |
| 4 | 1165 | 680 | 535 |  | 1390 | 590 | 1520 | 1907 | 2680 | 2671 | 1808 |
| 5 | 1663 | 1170 | 1025 | 1390 |  | 800 | 1730 | 2322 | 3112 | 3250 | 2298 |
| 6 | 863 | 370 | 225 | 590 | 800 |  | 930 | 1522 | 2312 | 2450 | 1498 |
| 7 | 945 | 1145 | 1155 | 1520 | 1730 | 930 |  | 780 | 1570 | 1903 | 1011 |
| 8 | 1137 | 1337 | 1372 | 1907 | 2397 | 1597 | 780 |  | 790 | 1745 | 1203 |
| 9 | 1890 | 2090 | 2125 | 2600 | 3150 | 2350 | 1570 | 790 |  | 1730 | 1948 |
| 10 | 1990 | 2190 | 2225 | 2677 | 3230 | 2430 | 2470 | 1745 | 1730 |  | 1039 |
| 11 | 1038 | 1238 | 1273 | 1808 | 2298 | 1498 | 1518 | 1755 | 1948 | 1039 |  |

Table 3
The Savings Table

| $j$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 |  | 1063 | 1058 | 1066 | 1066 | 373 | 373 | 373 | 373 | 373 |
| 3 | 1063 |  | 1285 | 1293 | 1293 | 420 | 420 | 420 | 420 | 420 |
| 4 | 1058 | 1285 |  | 1438 | 1438 | 545 | 395 | 395 | 478 | 395 |
| 5 | 1060 | 1293 | 1438 |  | 1726 | 833 | 478 | 441 | 403 | 403 |
| 6 | 1066 | 1293 | 1438 | 1726 |  | 833 | 478 | 441 | 403 | 403 |
| 7 | 373 | 445 | 590 | 878 | 878 |  | 1257 | 1220 | 927 | 927 |
| 8 | 373 | 420 | 395 | 403 | 403 | 1302 |  | 2237 | 1382 | 972 |
| 9 | 373 | 420 | 395 | 403 | 403 | 1265 | 2237 |  | 2450 | 980 |
| 10 | 373 | 420 | 478 | 403 | 403 | 465 | 1382 | 2150 |  | 1989 |
| 11 | 373 | 420 | 395 | 403 | 403 | 465 | 420 | 980 | 2841 |  |

associated with spot 3. Proceed in this manner and the final tram route for the Lamphun City tour is $1-2-3-6-4-5-7-8-9-10-11-1$ with a total distance of 10,050 meters.

As for applying the savings algorithm to solving this tram routing problem, first the so-called savings table must be constructed. Its $s_{i j}$ entry is the distance saved from going from Spot $i$ to Spot $j$ via Spot $k$ according to this formula $s_{i j}=d_{i, k}+d_{k, j}-d_{i, j}$ where $d_{i j}$ is the distance from Spot $i$ to $j$ in Table 1.

The savings table is shown in Table 3. Select the overall largest value, that is, the most saved distance. In this case, traveling from Spot 11 to Spot 10 saves 2,841 meters. Then, cross off row 11 and column 10 . Next, look at where to go next, either into Spot 11 or out of Spot 10. The most distance saved is 2150 , out of Spot 10 into Spot 9.

Continue doing in this manner and the final tram routes obtained from this savings algorithm are $1-11-10-9-8-7-6-5-4-3-2-1$ or $1-11$ $-10-9-8-7-5-6-4-3-2-1$ with a tied total distance of 9,770 meters.

Step 5 (Result Comparison): For comparison purposes, the total distance of the current tram route is tabulated along with those of the routes obtained from the LP model, the nearest neighbor and the savings algorithms in Table 4. Also, the distances reduced from the three methods, when compared to that of the current route are also shown in Table 4, both in meters and percentage.

It can be seen that the total distance from the LP model and that of the savings algorithm are tied at the minimum of 9,770 meters. Therefore, the optimal tram route for the Lamphun City tour can be either one of the following: 1-2-3-4-6-5-7-8-9-10-11-1 from the LP model, or

Table 4
Comparison Table

|  | Total Distance <br> (meters) | Distance Reduced <br> (meters) | Distance Reduced <br> (\%) |
| :--- | :---: | :---: | :---: |
| Current route | $\mathbf{1 0 , 7 7 9}$ | - | - |
| LP Model | 9,770 | 1,009 | $9.36 \%$ |
| Nearest Neighbor | 10,050 | 729 | $6.76 \%$ |
| Savings Algorithm | 9,770 | 1,009 | $9.36 \%$ |

$1-11-10-9-8-7-6-5-4-3-2-1$ or $1-11-10-9-8-7-5-6-4$ $-3-2-1$ from the savings algorithm.

## 5. Conclusion

This research studies the tram routing problem for the Lamphun City tour. First, the data regarding the locations of the tourist spots that the tram has to visit as well as intersections are gathered with traveling distances from one to another. The shortest path problem is then used in finding the direct shortest distances between the tourist spots.

Formulated as a linear programming (LP) model for the traveling salesman problem (TSP), this tram routing problem can then be solved by Excel solver to obtain an optimal route of 9,770 meters. In addition, another two algorithms, namely, the nearest neighbor and the savings algorithm, are employed on the problem. Only the two routes resulted from the savings algorithm yield the same minimum distance of 9,770 meters as that of the LP-model route.

In conclusion, the current tourist tram route for the Lamphun City of 10,779 meters can be reduced to 9,770 meters using one of the optimal routes obtained in this research. If the results are implemented, savings on both managerial and energy costs and traveling time will be definitely assured. Thus, this will bring in more revenue to the city and simultaneously draw more tourists to the City of Lamphun.

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